

1912  
St67

STOKES & VANIMAN

Hunting of Synchronous Machines

Electrical Engineering

B. S.

1912




THE UNIVERSITY  
OF ILLINOIS  
LIBRARY

1912  
5667







Digitized by the Internet Archive  
in 2014

<http://archive.org/details/huntingofsynchro00stok>



# HUNTING OF SYNCHRONOUS MACHINES

BY

JOHN WILLIAM STOKES  
AND  
ROY L VANIMAN

---

THE S I S

FOR THE

DEGREE OF BACHELOR OF SCIENCE

IN

ELECTRICAL ENGINEERING

---

COLLEGE OF ENGINEERING

UNIVERSITY OF ILLINOIS

1912

1912  
ست 67

UNIVERSITY OF BIRMINGHAM

UNIVERSITY OF ILLINOIS

May 30, 1902

THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

JOHN WILLIAM STOKES AND ROY L VANIMAN

ENTITLED HUNTING OF SYNCHRONOUS MACHINES

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF BACHELOR OF SCIENCE IN ELECTRICAL ENGINEERING

W. F. Schaller

Instructor in Charge

APPROVED:

E. J. Berg

HEAD OF DEPARTMENT OF ELECTRICAL ENGINEERING.





## C O N T E N T S

|  | Page |
|--|------|
| I Introduction . . . . .   | 1    |
| II Explanation of Phenomenon . . . . .   | 1    |
| III Analogy Drawn Between the Oscillations of the<br>Rotating Element and the Vibrations of a<br>Damped Spring . . . . . | 3    |
| IV Derivation of Equation for Free Oscillations . . .  | 3    |
| V Derivation of Damping Constant K . . . . .   | 6    |
| VI Numerical Example . . . . .   | 8    |
| VII Negative Damping . . . . .   | 11   |
| VIII Discussion of Equation for Free Oscillation . . .   | 13   |
| IX Remedies for Hunting . . . . .  | 14   |
| X Curves Showing Variation of Beats per Minute With<br>Air-Gap, Excitation, Moment of Inertia and Load                   | 14   |
| XI Derivation of Equation for Forced Oscillations . .  | 15   |
| XII Discussion of Equation for Forced Oscillations . .   | 18   |
| XIII Conclusion . . . . .  | 19   |



## HUNTING OF SYNCHRONOUS MACHINES

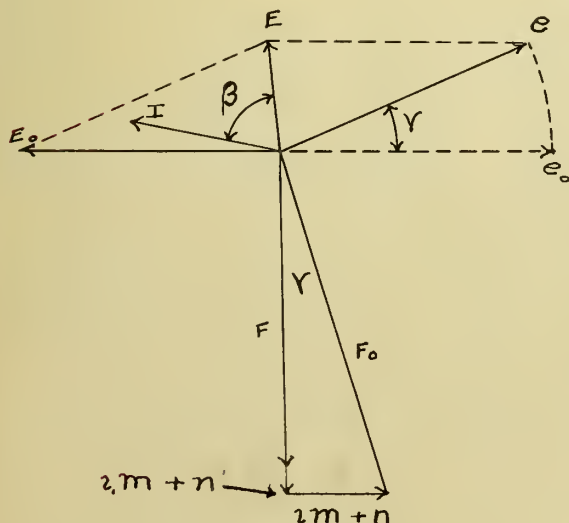
### I Introduction

This thesis is an elementary treatise on the hunting of synchronous machines. The equations deduced are based on an analogy drawn between the oscillations of the armature of the machine and the vibrations of a damped spring.

### II Explanation of Phenomenon

Hunting in synchronous machines is a phenomenon in which the armature or rotating part of the machine develops oscillations about the state of steady motion. It is not difficult to see how such oscillations or fluctuations may be started. Consider a generator supplying power to a synchronous motor and that the generator runs at absolutely constant speed, so that the frequency of the e.m.f. <sup>or</sup> speed of the vector  $E_0$  of the generator is constant.

Figure 1.



When the motor is running at constant speed and the load is constant, the angle  $\gamma$  as shown in Figure (1) remains constant. Suppose that a sudden change of load takes place, say an increase. A momentary retardation of the motor results and this retardation will continue until the increase in driving torque or power output, due to increase of  $\gamma$ , (since output increases nearly as  $\gamma$ ) becomes





equal to the increase in the resisting torque, due to the larger load. When the retardation ceases, the motor is running at a slower speed than that of synchronism. Hence  $\gamma$  will go on increasing; the driving torque will increase above the value required to overcome the resisting torque, acceleration will take place and the speed of the motor will increase. When the speed of synchronism is reached  $\gamma$  will cease to increase, and will then begin to decrease, since acceleration is still taking place the vector  $e$  as shown in Figure(1) will gain on  $E_0$ . As  $\gamma$  decreases the driving torque decreases, and at a certain stage becomes equal to the resisting torque. Acceleration now ceases; but at this point the motor is running at a speed above synchronism, so that  $\gamma$  will go on decreasing and the driving torque decreasing: retardation takes place and synchronous speed is reached when  $\gamma$  becomes a minimum. Further retardation now takes place,  $\gamma$  begins to increase in opposite direction, and so on.

A sudden change of load is thus seen to start oscillations of the rotating parts of the machine which become superposed upon the uniform rotation. There are, however, other ways in which such oscillations may be started. Let, for instance, the load remain constant and the speed of the generator undergo a slight change, say an increase. This corresponds in the figure to an advance of the vector  $E_0$  towards  $e$  (or an increase of  $\gamma$ ) and it is evident from what has been said that this will start oscillations. A sudden change in exciting current of either generator or motor will produce similar results.





### III Analogy Drawn Between the Oscillations of the Rotating Element and the Vibrations of a Damped Spring

The physical conception of hunting in synchronous motors as described above is similar in every respect to the hunting which takes place in parallel operation of alternators, in rotary converters, in induction motors and generators, and in fact in all synchronous machinery. The oscillation of the armature about the state of steady motion is analogous to the oscillation of a spring when it vibrates against the resistance of a dash-pot. It is thus seen that the motion is that of a damped oscillation and it may be represented by the following law or equation:

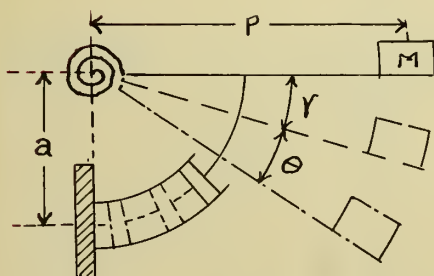
$$J \frac{d^2\theta}{dt^2} + K \frac{d\theta}{dt} + A\theta = 0 \quad (1)$$

in which  $J$  is the moment of inertia of the revolving element,  $K$  a constant depending on the damping effects, and  $A$  the torque per unit angular displacement of the armature from its state of steady motion, while  $\theta$  is this displacement.

### IV Derivation of Equation for Free Oscillations

By reference to Figures 2 and 3 it is possible to explain the derivation of the coefficients in equation (1) and the re-

Figure 2.



lations which obtain in the analogy between the damped oscillations of the spring and those of the rotating elements of the synchronous machine about the state of steady motion.

Let  $M$  = Mass, which is analogous to mass of rotating elements.









$$T = \frac{33000P}{2\pi N} \text{ in ft. lbs. where } N = \text{R.P.M.}$$

$$A = \frac{T}{Y} = \frac{\frac{33000P}{2\pi N}}{Y} = \frac{33000P}{2\pi N Y}$$

Equation (1) is integrated by placing

$$\theta = \epsilon^{mt}$$

$$\frac{d\theta}{dt} = m\epsilon^{mt}$$

$$\frac{d^2\theta}{dt^2} = m^2\epsilon^{mt}$$

Substituting the above values in the differential equation and simplifying  $m^2 + \frac{K}{J}m + \frac{A}{J} = 0$

$$m = -\frac{K}{2J} \pm \sqrt{\frac{K^2}{4J^2} - \frac{A}{J}}$$

$$m_1 = -\frac{K}{2J} + \sqrt{\frac{K^2}{4J^2} - \frac{A}{J}} \quad \text{and} \quad m_2 = -\frac{K}{2J} - \sqrt{\frac{K^2}{4J^2} - \frac{A}{J}}$$

The complete solution of the equation is represented by

$$\theta = C_1\epsilon^{m_1t} + C_2\epsilon^{m_2t} \quad (2)$$

$$= \epsilon^{-\frac{Kt}{2J}} \left[ C_1\epsilon^{jt\sqrt{\frac{A}{J} - \frac{K^2}{4J^2}}} + C_2\epsilon^{-jt\sqrt{\frac{A}{J} - \frac{K^2}{4J^2}}} \right] \quad (3)$$

$$= A_1\epsilon^{-\frac{Kt}{2J}} \sin \left[ t\sqrt{\frac{A}{J} - \frac{K^2}{4J^2}} + \beta \right]$$

which is the solution if  $m_1$  and  $m_2$  are imaginary. The constants  $A_1$  and  $\beta$  may be determined if initial conditions are known. Since  $K \frac{d\theta}{dt}$  is a torque resisting the motion it may be considered that  $K$  is composed of two quantities, one a damping effect due to eddy currents in the pole faces and currents in the amortisseur (by amortisseur is meant the squirrel cage winding in the pole faces of the machine), and the other a damping effect due to the shifting of the magnetic flux, by variations of load. This last phenomenon would occur only in extreme cases and in the following derivation of  $K$  the damping effect due to the amortisseur winding only will



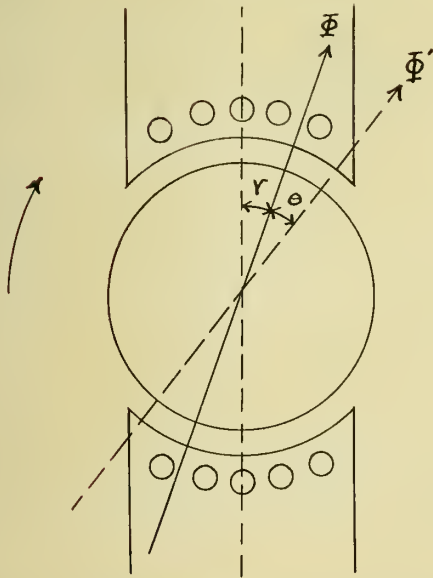


be considered.

### V Derivation of Damping Constant K

Let Figure 4 represent a two-pole machine running under load and consider that the vector  $\Phi$  represents the position of the

Figure 4



effective flux per pole through the machine, for steady motion. If oscillations are set up the flux will be shifted back and forth and thus induce an e.m.f.  $e$  in the amortisseur bars. This e.m.f. will send a current through the amortisseur which will result in a dissipation of energy. This dissipation of energy represents the energy due to damping effects resisting the oscillation (Newton's

Third Law of Motion). In this derivation it is assumed that the oscillations of flux are in phase with the oscillations of m.m.f. and that the variation of effective flux is negligible.

Let  $\Phi$  = effective flux per pole.

$l$  = length of amortisseur bars which cut flux in cm.

$\theta$  = displacement of flux from steady running condition.

$\beta$  = density in lines per sq. cm.

$a$  = radius of amortisseur winding in cm.

$a \frac{d\theta}{dt} \cos \theta$  = linear velocity of cutting lines of force by bars.

$a \frac{d\theta}{dt} \cos \theta \beta l$  = e.m.f. induced per bar in abvolts.

Since  $\theta$  in mechanical measurements equals  $2 \times \theta$  in



electrical measurements divided by the number of poles,  $\theta$  is very small and  $\cos \theta$  may be placed equal to unity in the expression for e.m.f. Then

$$e = a\beta l \frac{d\theta}{dt} \times 10^{-8} \text{ in volts}$$

Neglecting the reactance of the amortisseur which is small for low frequencies

$$\begin{aligned} \frac{e^2}{R} &= \text{watts consumed per pole by damping} \\ &= \frac{a^2 \beta^2 l^2 \left(\frac{d\theta}{dt}\right)^2 \times 10^{-16}}{R} \end{aligned}$$

where  $R$  = resistance of amortisseur circuit per pole.

Let  $Fa = K \frac{d\theta}{dt}$  = resisting torque due to damping effect.

$$Fa \frac{d\theta}{dt} = K \left(\frac{d\theta}{dt}\right)^2 = \frac{e^2}{R}$$

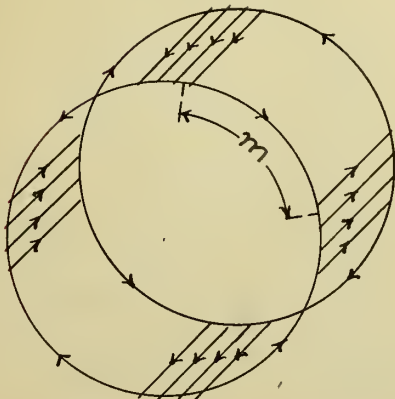
$$\text{With } F \text{ in lbs. } \frac{Fa \frac{d\theta}{dt}}{30.4} \times \frac{746}{550} = \frac{a^2 \beta^2 l^2 \left(\frac{d\theta}{dt}\right)^2 \times 10^{-16}}{R}$$

$$\frac{Fa}{30.4} = \frac{.74a^2 \beta^2 l^2 \frac{d\theta}{dt} \times 10^{-16}}{R} = K \frac{d\theta}{dt}$$

Where  $\frac{Fa}{30.4}$  = torque in lbs. ft.

$$\text{The } K = \frac{.74a^2 \beta^2 l^2 \times 10^{-16}}{R}$$

Figure 5



Assuming that the current will take the path as shown in Figure 5.

Let  $m$  = resistance of ring as shown.

$s$  = resistance of bar as shown.

$N$  = number of bars per pole

$$\text{then } R = \frac{2mN + 4s}{N}$$

Since there are two e.m.f.s  $e$  in series tending to send current





through the circuit  $\frac{2e}{R}$  = current flowing

$\frac{4e^2}{R}$  = watts lost per circuit, and finally

$$K/\text{pole} = \frac{2.97a^2\beta^2l^2 \times N \times 10^{-16}}{2mN + 4s} \quad (6)$$

The expression for K as shown is a constant since it is composed only of constant terms determined by the design of the machine.

### VI Numerical Example

Given a machine of the following rating: A. T. B. - 12 poles - 300 K.W. - 600 R. P. M. - 2300 volts - Round rotor.

Weight of revolving element = 1500 lbs.

radius of gyration = 1.3 ft.

$$\frac{1500}{32.2} \times 1.3^2 = 78.8 = J \text{ in gee lb. units}$$

$\beta$  = 60,000 lines per sq. in.

= 9,300 lines per sq. cm.

diameter of amortisseur = 36 in. and  $a$  = 1.5 ft.

pole arc = .65 of pole pitch.

$$\frac{\pi \times 36 \times .65}{12} = 6.1 \text{ in. periphery of pole shoe.}$$

$$\frac{58}{6.1} = 10 \text{ in.} = l \text{ where } 58 \text{ sq. in.} = \text{area of pole face}$$

Assume 4 bars per pole in amortisseur

#0000 B & S copper wire equals size for bars

#0000 B & S brass equals size for rings

Assuming that the bars are 13 in. long and that the length of ring considered in the circuit is 9 in.: then  $s$  = .00005 ohms, and  $m$  = .00012 ohms.

and  $2mN + 4s$  = .00116 ohms.



$$K = \frac{2.97 \times 1.5 \times 30.4^2 \times 9300^2 \times 25.4^2 \times 4 \times 10^{-16}}{.00116} = 120$$

$$120 = K \text{ per pole}$$

$$K \text{ for machine} = 12 \times 120 = 1440$$

For full non-inductive load on the machine the angle with .32 in. air-gap is 20 electrical degrees.

$$\frac{20 \times 2}{12 \times 57.3} = .058 \text{ radians mechanical displacement from zero}$$

$$\text{load position. } \frac{2\pi NT}{33000} = \frac{300}{.746} \text{ H.P.}$$

$$6.28 \times 600 \times T = \frac{300}{.746} \times 33000$$

$$T = 3500 \text{ lbs. ft.}$$

$$A = \frac{T}{Y} = \frac{3500}{.058} = 60,000 \text{ lb. ft. per radian displacement of}$$

armature.

$$\sqrt{\frac{A}{J} - \frac{K^2}{4J^2}} = \sqrt{\frac{60000}{78.8} - \frac{1440^2}{2 \times 78.8^2}} = 27.4$$

Substituting the constants calculated in the equation for  $\theta$

$$\theta = A_1^{-9.15t} \sin(27.4t + \beta) \quad (7)$$

$$27.4t = 2\pi ft \quad f = 4.35 \text{ beats per second}$$

The constants  $A_1$  and  $\beta$  are determined when the conditions under <sup>which</sup> the oscillations take place are known.

Consider, for instance, that an alternator is carrying a certain load and that this load be thrown off. Referring to the vector diagram, it is seen that the machine when loaded has the armature displaced from the no-load position by the angle  $Y$  which for full non-inductive load on the alternator given is .058 radians. When the load is thrown off the armature oscillates back and forth through the no-load position. Under these









$$\text{tangent } 27.4t = 1.32$$

$$27.4t = 52^\circ - 50' = .922 \text{ radians}$$

$$t = .0336 \text{ radians.}$$

Substituting value of  $t$  in expression for  $\frac{d\theta}{dt}$

$$\begin{aligned} \frac{d\theta}{dt} &= -e^{-.307} [.53 \times .604 + 1.59 \times .797] \\ &= -1.183 \text{ radians per second} \end{aligned}$$

$$K \frac{d\theta}{dt} = \text{Torque in lbs. ft.}$$

$1.183 \times 1440 = 1690 \text{ lbs. ft.} = \text{Maximum torque due to amortisseur.}$

$$\begin{aligned} e &= 1.5 \times 30.4 \times 25.4 \times 9300 \times 1.183 \times 10^{-8} \\ &= .128 \text{ volts maximum e.m.f. induced per bar.} \end{aligned}$$

$$R = \frac{2mN + 4s}{N} = .0003 \text{ ohms}$$

$$i = \frac{2 \times .128}{.0003} = 854 \text{ amperes maximum current in amortisseur.}$$

$2ei = 2 \times .128 \times 854 = 218 \text{ watts maximum power consumed in amortisseur per pole.}$

$$218 \times 12 = 2620 \text{ total maximum watts consumed.}$$

$$K\left(\frac{d\theta}{dt}\right)^2 = 1440 \times \frac{2}{1.183} = 2000 \text{ ft. lbs. per second maximum total power consumed}$$

$$2000 \times \frac{746}{550} = 2620 \text{ watts which check with above value calculated from maximum } e \text{ and } i.$$

## VII Negative Damping.

As previously mentioned in this discussion the damping effects may be negative thus giving a positive exponential and under these conditions if oscillations are once set up they will

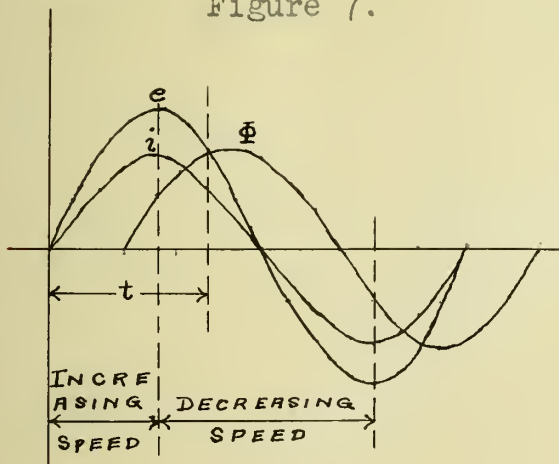




continue to increase in amplitude until the angle  $\gamma + \theta$  is greater than the tangent  $-1 \frac{X}{R}$  (where  $X$  is the synchronous reactance of the machine and  $R$  is the resistance) in which case the machine will fall out of step. This negative damping effect may be found external to the machine or internal in its magnetic circuit. Negative damping effects due to external sources may result from the magnetic field of a self-excited direct current generator

driven by a synchronous motor.

Figure 7.



Considering Figure 7, the axis of the curves represents conditions for steady operation or the constant values of induced e.m.f., current, and flux of the direct-current generator. If variations of speed are impressed upon the generator the conditions will be as shown in Figure

7, assuming simple harmonic variations above and below normal speed. It is seen from the curves for  $\Phi$  and  $i$  that the values for flux and current are both above normal even though the speed is decreasing. The increased torque during decrease of speed will give a negative damping producing the same effect as a positive force applied to a pendulum at the moment its acceleration changes from negative to positive.

The change of power input into a synchronous motor with change of speed may cause the governor to act on the prime mover driving the generator which supplies power to the motor. Due to the sluggish action, or in some cases to the extreme sensitiveness of the governor it will lag behind the change in output of the



prime mover giving a pulsation of the generator frequency or e.m.f. It is evident that as the speed of the motor decreases, the generator speed decreases also, the motor will take less power and the speed will decrease more and more.

In a synchronous motor the effective flux through the machine is that which gives the counter e.m.f. plus the IR and IX drops (where R. is the resistance of the armature and X is self inductive reactance). This flux corresponds to the flux sent through the machine by the total m.m.f. of the field winding minus the m. m. f. of armature reactions, which latter vary in intensity and in phase during the oscillation of  $\phi$  from  $\gamma + \theta$  to  $\gamma - \theta$  and so forth. While the counter e.m.f.  $e$  is constant, the magnetic flux is not (the variations of counter e.m.f.  $e$  and generated e.m.f.  $e$  being compensated for by the variation of IR and IX drop in the motor.), but pulsates with the oscillations of the machine. Owing to the fact that the change of flux lags behind the change of m.m.f. for reasons previously mentioned, the pulsations of the flux will be such as to induce a negative damping effect just at the time when the oscillation of the machine has reached its maximum position and started back, and so on.

#### VIII Discussion of Equation for Free Oscillations.

From the above discussion it is seen that the hunting of synchronous machines due to free oscillations alone resolves itself into an investigation of the damping effect, or the term  $K$  in the equation.

If  $\frac{K^2}{4J^2}$  is greater than  $\frac{A}{J}$  the frequency term  $\sqrt{\frac{A}{J} - \frac{K^2}{4J^2}}$  will be imaginary and the machine will not oscillate. If  $K$  is





negative and  $\frac{A}{J}$  is greater than  $\frac{K^2}{4J^2}$  the motor will oscillate with constantly increasing amplitude until  $\gamma + \theta$  equals  $\tan^{-1} \frac{X}{R}$  when the machine will fall out of step. On the other hand, if  $K$  is positive, the motor will oscillate with constantly decreasing amplitude, finally reaching a stage of steady motion.

### IX - Remedies for Hunting

The causes of hunting due to free oscillations have been discussed and with the knowledge gained thereby as to its causes it is possible to give some methods of remedying it.

(a) An elimination of the negative damping effects by changing period of oscillation so as to change phase relation of flux and m.m.f.

(b) Sufficient damping effect by the use of amortisseur in the pole-faces.

(c) Variation of moment of inertia of revolving parts in order to change frequency of oscillations.

(d) Variation of length of air-gap and voltage to give change in frequency of oscillations.

### X - Curves Showing Variation of Beats per Minute with Air-gap, Excitation, Moment of Inertia and Load.

The data for the curves between beats per minute as ordinates and the other variables as abscissae is obtained by using the rating of the 300 K. W. alternator together with the constants of same, such as armature reaction, synchronous reactance, etc. The curves show very plainly the effect upon the beats per minute by varying the length of air-gap, the excitation, the load, and the moment of inertia.



560 BEATS/MINUTE

480

400

320

240

160

80

0

0.10

0.20

0.30

0.40

0.50

LENGTH OF AIRGAP - INCHES

CURVE SHOW-  
ING RELATION BETWEEN  
BEATS/MINUTE AND —  
LENGTH OF AIRGAP FOR  
AN ALTERNATOR OF RAT-  
ING - A.T.B. - 12 POLES -  
300 K.W. - 600 R.P.M.<sup>9</sup><sub>N</sub>  
2300 VOLTS. - J = 18.8  
ROUND ROTOR





280 BEATS/MINUTE

260

240

220

200

180

160

140

 $1\frac{1}{2}$  LOAD

FULL-LOAD

 $\frac{1}{2}$  LOAD $\frac{1}{4}$  LOAD

CURVES SHOW-  
ING RELATION BETWEEN  
BEATS/MINUTE AND EX-  
CITATION FOR AN ALTER-  
NATOR OF RATING -  
A.T.B.-12 POLES-300 K.W.  
600 R.P.M.-2300 VOLTS.  
ROUND ROTOR.  $J=18.8$   
AIR GAP = 0.16"

EXCITATION-VOLTS.

1800

2000

2200

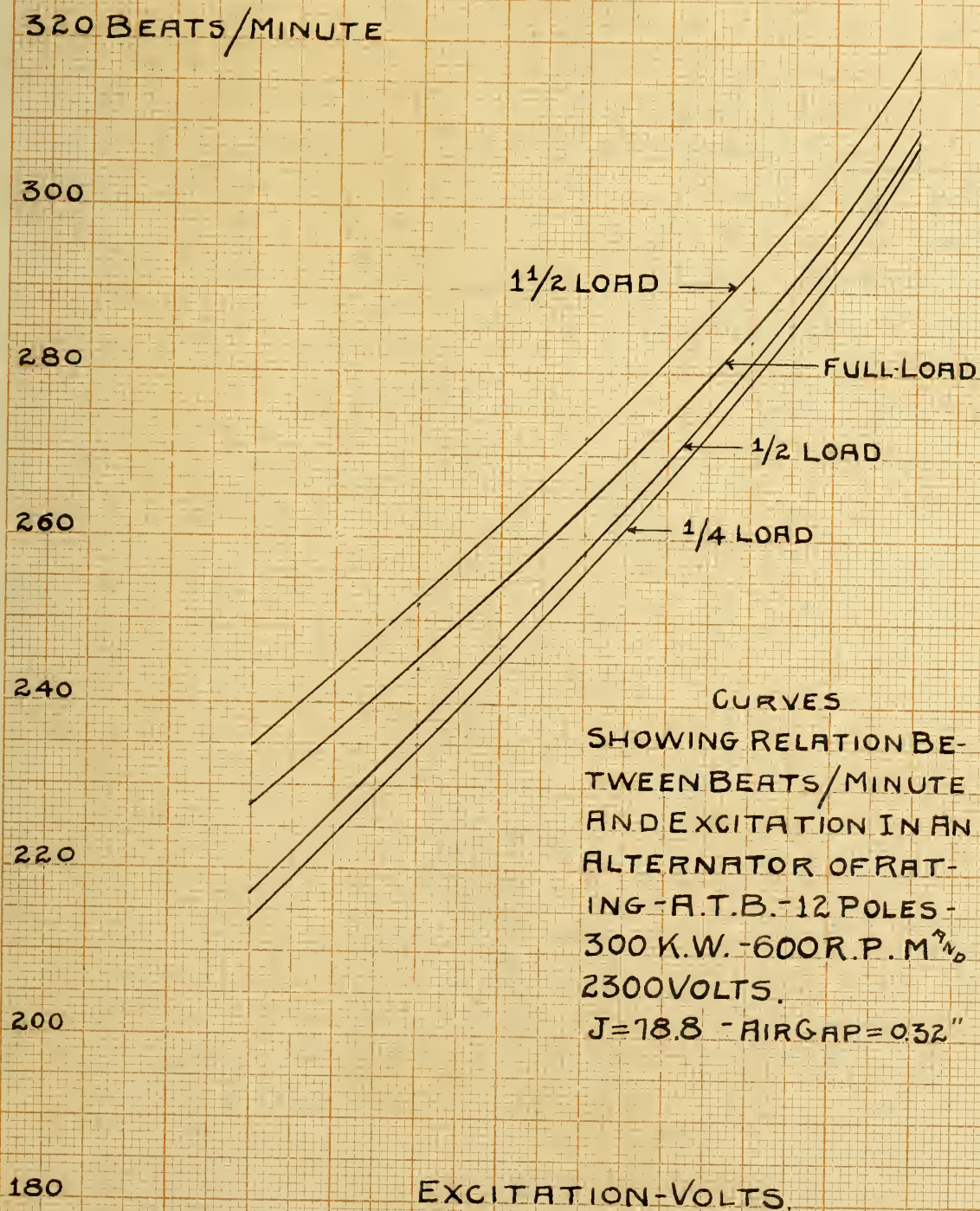
2400

2600

2800











460 - BEATS/MINUTE

420

380

340

300

260

220

180

140

CURVE SHOW-  
ING RELATION BETWEEN  
BEATS/MINUTE AND MOM-  
ENT OF INERTIA OF RO-  
TATING PARTS FOR AN  
ALTERNATOR OF RATING  
A.T.B. - 12 POLES - 300 K.W.  
600 R.P.M. - 2300 VOLTS.  
ROUND ROTOR  
AIR GAP = 0.32"

J - GEE-POUND UNITS.

0

20

40

60

80

100

120



# XI Derivation of Equation for Forced Oscillations

The forces tending to cause free oscillations of the rotating elements are the effective forces  $J \frac{d^2\theta}{dt^2}$ , the impressed forces of restitution  $\frac{A}{J} \theta$  and the resisting forces  $K \frac{d\theta}{dt}$ . The oscillations may also be subjected to the action of a periodic force which prevents the vibrations dying away. This is called the external force. It is represented by the addition of a term  $f(t)$  to the right-hand side of the regular equation of motion, so that

$$\frac{d^2\theta}{dt^2} + \frac{K}{J} \frac{d\theta}{dt} + \frac{A}{J} \theta = f(t) \quad (10)$$

The above equation is the one for forced oscillations and may be applied to the hunting of synchronous machines in which case there is a periodic disturbing effect represented by  $f(t)$ .

The effective force and the three kinds of impressed forces all produce their own effects, and each force is represented in the equation of motion by its own special term;

$J \frac{d^2\theta}{dt^2}$  for effective forces

$K \frac{d\theta}{dt}$  for resisting forces

$A\theta$  for forces of restitution

$K_1 \cos nt$  for periodic disturbing forces

For convenience the above differential equation may be written

$$\frac{d^2\theta}{dt^2} + 2f \frac{d\theta}{dt} + q^2\theta = K_0 \cos nt \quad (11)$$

in which case  $2f$  is equal to  $\frac{K}{J}$ ;  $q = \sqrt{\frac{A}{J}}$  and  $K_0 = \frac{K_1}{J}$ .

The complete solution of equation (11) consists of a complimentary function and a particular integral which are superposed or added together. The complimentary function may be ob-





tained by solving the equation

$$\frac{d^2\theta}{dt^2} + 2f \frac{d\theta}{dt} + g^2\theta = 0 \quad (12)$$

and gives the oscillations of the system when not influenced. This integral, as mentioned previously, represents the free or natural oscillations. The particular integral represents the effects of the periodic impressed forces which produce the forced oscillations.

Let equation (11) indicate the motion or oscillations of a synchronous machine acted upon by a force which is a simple harmonic function of the time  $t$ . The solution of the complimentary function is

$$\theta = A_1 e^{-ft} \sin [t\sqrt{g^2 - f^2} + \beta] \quad (13)$$

or the expression for free damped oscillations. Any particular integral represents the forced vibrations, but there is one particular integral which is more convenient than any other, viz.

$$\theta = A \cos nt + B \sin nt \quad (14)$$

The complimentary function contains the two arbitrary constants which are necessary to define the initial conditions; consequently the particular integral needs no integration constant. It is now necessary to determine the forced oscillation due to the periodic force and evaluate the constants  $A$  and  $B$  in the equation (14)

First, differentiate equation (14) and substitute results in equation (11) from which result two equations

$$\frac{d\theta}{dt} = -An \sin nt + Bn \cos nt \quad (15)$$

$$\frac{d^2\theta}{dt^2} = -An^2 \cos nt - Bn^2 \sin nt \quad (16)$$





$$-An^2 \cos nt - Bn^2 \sin nt + 2f (-An \sin nt + Bn \cos nt) + q^2 (A \cos nt + B \sin nt) = K_0 \cos nt \quad (17)$$

Equating coefficients of  $\cos nt$  on each side of the equation (17) and also of  $\sin nt$  there results two equations in A and B, as follows

$$-An^2 + 2Bfn + q^2 A = K_0 \quad \text{and} \quad (18)$$

$$-Bn^2 - 2Afn + q^2 B = 0 \quad (19)$$

Solving for A and B

$$A = \frac{K_0(q^2 - n^2)}{(q^2 - n^2)^2 + 4f^2 n^2} \quad \text{and} \quad B = \frac{2K_0 fn}{(q^2 - n^2)^2 + 4f^2 n^2}$$

Dividing A and B by  $\frac{K_0}{\sqrt{(q^2 - n^2)^2 + 4f^2 n^2}} = R = \frac{K_0}{J \sqrt{\left(\frac{A}{J} - n^2\right)^2 + \frac{K^2}{J^2} n^2}}$

There results  $A = R \cos \alpha$  and  $B = R \sin \alpha$  where tangent  $\alpha =$

$$\frac{2fn}{q^2 - n^2} = \frac{\frac{K}{n} - \frac{n}{J}}{\frac{A}{n} - \frac{n}{J}} \quad \text{and finally}$$

$$\theta = R \cos (nt - \alpha) \text{ for the particular integral.}$$

The above expression for  $\theta$  indicates the forced oscillations of the machine which are due to the periodic force  $K_0 \cos nt$ . The forced oscillation is not in phase with the principal oscillation induced by the effective force but lags behind by a definite amount  $\alpha$ .

Since the complete solution of the equation (11) is the sum of the particular integral and the complimentary function

$$\theta = A_1 e^{-ft} \sin qt + R \cos (nt + \alpha) \quad (20)$$

The integration constant  $A_1$  may be evaluated when the initial conditions are known. If the machine is not oscillating when the force begins to act, then  $t = 0$ .

Substituting this value of  $t$  in equation (20) and solving for  $A_1$



$$A_1 = \sqrt{R^2 \cos^2 \alpha + R^2 \left( \frac{n}{q} \sin \alpha + \frac{2f}{q} \cos \alpha \right)^2}.$$

## XII - Discussion of Equation for Forced Oscillations

At the beginning, therefore the amplitude of the free vibrations is of the same order of magnitude as the forced oscillations. If the damping  $2f$  is small and  $n$  is nearly equal to  $q$  the damping factor  $e^{-ft}$  will be nearly unity,  $\alpha$  is nearly  $\frac{\pi}{2}$  and  $A_1$  is equal to  $R$ .

Then  $\theta = R (\sin nt - \sin qt)$

$R$  always has the same sign whatever the signs of  $n$  and  $q$ ;  $2f$  is positive hence  $\sin \alpha$  is positive and the angle  $\alpha$  lying in the first two quadrants, ranges from 0 to  $\pi$ . On the other hand the sign of  $\cos \alpha$  does depend on the relative magnitudes of  $n$  and  $q$ ; if  $q$  be greater than  $n$ ,  $\alpha$  is in the first quadrant; if  $q$  is less than  $n$ ,  $\alpha$  is in the second quadrant and if  $q$  equals  $n$ ,  $\alpha = \frac{\pi}{2}$ . The amplitude  $R$  of the forced vibrations is proportional to the intensity of  $K$ , of the external force. If  $f$  be very small  $R = \frac{K_0}{q^2 - n^2} = \frac{K}{A - Jn^2}$ ; in that case the more nearly  $q$  approaches  $n$  the greater will be the amplitude  $R$ . When  $q$  is equal to  $n$  then  $R = \frac{K_0}{2nf} = \frac{2K_1}{nK}$  so that the magnitude of  $R$  is conditioned by the damping constant. If  $f = 0$  and  $q = n$  then the machine is acted upon by the periodic force in step with the free oscillations, the amplitude of the forced vibrations will increase indefinitely and the machine will "fall out of step."

If the oscillations are strongly damped, the maximum amplitude does not occur when  $n = q$  but when  $\sqrt{(q^2 - n^2)^2 + 4f^2 n^2}$  is minimum. If  $n$  be variable the expression under the root sign is a minimum when  $n^2 = q^2 - 2f^2$  and  $R$  is maximum. It is thus seen





in case of forced oscillations that  $f$  should be large and  $q$  and  $n$  unequal in order to avoid troublesome cases of hunting.

### XIII Conclusion

Although the equations deduced are approximate, the results conform sufficiently to facts to warrant the following conclusions. From the discussion of hunting of synchronous machines as regards both free and forced oscillations of the rotating parts, it is seen that;

(a) hunting due to free oscillations is never serious except in cases of negative damping effect;

(b) this damping does not effect the beats per minute to any appreciable extent;

(c) the torque per unit angular mechanical displacement of the armature and the moment of inertia are the governing factors in the frequency of the oscillations and;

(d) the torque per unit angular displacement is dependent on the excitation and length of air-gap, and is nearly independent of the load. For forced oscillations, if the period of the disturbing force is equal or nearly equal to the period of the free oscillations, the hunting will be serious unless there is a heavy damping effect. With this heavy damping effect, the loss in the amortisseur winding will decrease the efficiency of the machine considerably. The damping effect is the controlling factor in the amplitude of the forced oscillations, when the period of the disturbing force is very nearly equal to that of the free oscillations.











UNIVERSITY OF ILLINOIS-URBANA



3 0112 082197614